$$\int \tan^4 x \, dx \to \int \tan^2 x \cdot \tan^2 dx \to \int \tan^2 x \left(\sec^2 x - 1\right) dx$$

$u = \tan x \quad du = \sec^2 x \, dx$

$$\int \left(\tan^2 x \sec^2 x - \tan^2 x\right) dx$$

$$\int \tan^2 x \sec^2 x \, dx - \int \tan^2 x \, dx$$

$$\int u^2 \, du \qquad \int \left(\sec^2 x - 1\right) dx$$

$$\frac{u^3}{3} + C \qquad \int \sec^2 x \, dx - \int 1 \, dx$$

$$\frac{\tan^3 x}{3} - \left(\tan x - x + C\right)$$

$$\int_0^1 \frac{10\sqrt{x}}{(1+x^{3/2})^2}\,dx \qquad u = 1 + x^{3/2},\quad du = \tfrac{3}{2}x^{1/2}$$

$$\int_0^1 \frac{10\sqrt{x}}{u^2}\,dx \longrightarrow \frac{20}{3}\int_0^1 \frac{du}{u^2} \Rightarrow \frac{20}{3}\int_0^1 \frac{1}{u^2}\cdot du$$

$$\frac{20}{3}\int_0^1 \frac{1}{u^2}\cdot du \Longrightarrow \frac{20}{3}\int_0^1 u^{-2}\cdot du \Rightarrow \frac{20}{3}\ -u^{-1}$$

$$F(x) = \frac{20}{3} - u^{-1} + C$$

$$\frac{20}{3}\ -u^{-1} \Longrightarrow F(x) = \frac{20}{3}\cdot -(1+x^{3/2})^{-1} + C$$

$$\frac{20}{3}\cdot -(1+1^{3/2})^{-1} + C - \frac{-20}{3}\cdot(1+0^{3/2})^{-1} + C$$

$$\frac{20}{3}\cdot\frac{-1}{2} - \frac{-20}{3}\cdot\frac{1}{1}$$

$$\frac{-20}{6} - \frac{-20}{3}\left(\frac{2}{2}\right) \Longrightarrow \frac{-20}{6} - \frac{-40}{6}$$

$$\frac{40}{6} - \frac{20}{6} = \frac{20}{6} \Longrightarrow \boxed{\frac{10}{3}}$$

Separation of variables

Goal is to find $y =$

- Slope fields
- separation of variables

$$\frac{dy}{dx} = (xy)^2 \longrightarrow \frac{dy}{dx} = x^2 y^2$$

$$\frac{dy}{y^2} = x^2\,dx$$

$$\int \frac{dy}{y^2} = \int x^2\,dx \implies \int \frac{1}{y^2}\,dy = \int x^2\,dx$$

$$-y^{-1} + C_1 = \frac{x^3}{3} + C_2$$

$$-y^{-1} = \frac{x^3}{3} + C$$

$$y^{-1} = \frac{C - x^3}{3} \qquad C - \frac{x^3}{3}$$

$$y = \frac{3}{C - x^3} \qquad y = C - \frac{3}{x^3}$$

Try

ⓐ $\frac{dy}{dx} = \frac{2x}{y}$

$\int y\,dy = \int 2x\,dx$

$\frac{1}{2}y^2 = x^2 + c$

$y^2 - 2x^2 = c$

ⓑ $\frac{dy}{dx} = \frac{5x}{y}$

$\int y\,dy = \int 5x\,dx$

$\frac{1}{2}y^2 = \frac{5}{2}x^2 + C$

ⓒ $\frac{dy}{dx} = \sqrt{x}\,y$

$\int \frac{dy}{y} = \int \sqrt{x}\,dx$

$\downarrow$

$\ln|y| = \frac{2}{3}x^{3/2} + C$

$e^{\ln|y|} = e^{\frac{2}{3}x^{3/2} + C}$

$|y| = e^{2/3 x^{3/2} + C}$

$y = \pm e$

$y = \pm e^{\frac{2}{3}x^{3/2}} \cdot e^{C}$

$\cancel{y = \pm e^{2/3 x^{3/2}} \cdot C}$

$\boxed{y = \pm C e^{2/3 x^{3/2}}}$

ⓓ $\frac{dy}{dx} = ky$

$\int \frac{dy}{y} = k\,dx$

$\ln|y| = kx + C$

$\cancel{y = \pm e^{kx + C}}$

$\boxed{y = \pm C e^{kx}}$

HW

Sect. 6.4

#1-10(5), 11, 14, 15, 18, 19, 22, 23

Read rest of 6.4